

# Screening Masses in Gluonic Plasma

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Both electric and magnetic screening masses in a nonperturbative gluonic background are investigated using operator product expansion. The magnetic screening mass is found to agree with lattice results whereas the electric screening mass is somewhat smaller than the one found on the lattice.

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The long-range properties of a thermal system are characterized by screening masses or the inverse of equal-time correlation lengths. For a given theory the screening masses determine the infrared sensitivity of various thermodynamic quantities as well as the spectral properties of the system [1]. In QED the screening associated with the electric fields exhibits a non-vanishing Debye mass whereas that with the magnetic fields does not show up due to gauge invariance. This indicates that the longest length scale in hot QED plasma is dominated by magnetic fields. On the other hand, in QCD the scenario is far more complicated than in QED due to the gauge dependence of the chromo-electric and magnetic fields, which leads to subtleties in the calculations.

The structure of QCD, at least near to phase transition, seems more complex than one usually expects. Perturbative predictions are upset by the presence of strong non-perturbative effects [2]. The nonperturbative determination of screening masses has been performed in lattice QCD (LQCD) [3–5] with an appropriate gauge fixing. The data are consistent with an overall exponential behaviour for the electric screening function in all gauges whereas that for the magnetic sector involves a nontrivial behaviour [3, 6]. In perturbation theory the electric screening mass to the lowest order is obtained as  $m_D \sim gT$  (the strong coupling constant is  $\alpha_s = g^2/4\pi$  and  $T$  is the temperature), which falls short of the nonperturbative description. On the other hand, magnetic screening cannot be addressed in perturbation theory but one expects the magnetic mass to be generated ( $m_m \sim gm_D \sim g^2T$ ) nonperturbatively in the static sector [7, 8]. Nonetheless, perturbative methods could only be reliable for temperatures far above the critical temperature. Moreover, the perturbative power-counting hierarchy of scales  $m_D > m_m \sim g^2T$ , is doubtful close to the critical temperature. Effective models using Polyakov Loop correlation [9], dimensionally reduced QCD [10],  $\mathcal{N} = 4$  supersymmetric Yang-Mills theory [11] and AdS/QCD [12] were employed to analyze the screening masses from gauge invariant correlators.

We note that the contributions from the nonperturbative magnetic sector reveal a strong dependence on these correlation functions but provide very useful information after all.

It is widely accepted that the nonperturbative dynamics of QCD is signaled by the emergence of power corrections in physical observables. These nonperturbative corrections are introduced via non-vanishing vacuum expectation values of local quark and gluonic operators such as the quark condensate  $\langle \bar{\psi}\psi \rangle$  and the gluon condensate  $\langle G_{\mu\nu}^a G^{a\mu\nu} \rangle$ , which are also measured in LQCD [13]. This approach of the operator product expansion (OPE) has met noticeable success in QCD sum rule calculations at zero temperature [14] and the calculation of the  $N$ -point functions in QCD at zero temperature [15]. Unlike QCD sum rules, the condensates do not appear in a gauge invariant combination in QCD Green's Function [16]. In addition, there is an explicit dependence on the gauge fixing parameter ( $\xi$ ) in the Wilson coefficients. The OPE has also provided some insight on the nonperturbative features of QCD at finite temperature. In-medium nonperturbative chiral quark propagator [17], quark-photon vertex [18] and dilepton production rate [19] in presence of dimension four electric and magnetic condensates were investigated some time ago. Also attempts were made to extract a nonperturbative electric screening mass from the gluon propagator [20]. In this work we calculate for the first time the structure of the screening masses in a very comprehensive way using OPE that provides us the nonperturbative information to the infrared sensitivity of QCD, which in turn may help in constraining the various thermodynamic and spectral properties of high temperature QCD matter.

In QCD the gluon polarization operator is not transverse in general,  $P^\mu \Pi_{\mu\nu}(P) \neq 0$ . The most general tensorial structure of the in-medium gluon self-energy for an  $O(3)$  invariant gauge fixing condition can be written as [21]

$$\pi_{\mu\nu}(\omega, p) = \pi_l \mathcal{P}_{\mu\nu}^l + \pi_t \mathcal{P}_{\mu\nu}^t + \pi_m \mathcal{M}_{\mu\nu} + \tilde{\pi} \mathcal{L}_{\mu\nu}. \quad (1)$$

We work here in covariant gauge and also omit the color indices for brevity. The Lorentz invariant single particle energy and momentum are, respectively, given as  $\omega = u \cdot P$  and  $p = \sqrt{(u \cdot P)^2 - P^2}$  where  $u^\mu$  is the four velocity of the heat bath and  $P = (p_0, \vec{p})$ . The projection operators are defined as [21, 22]

$$\mathcal{P}_{\mu\nu}^l = \frac{P^2}{\widetilde{P}^2} \bar{u}^\mu \bar{u}^\nu, \quad \mathcal{P}_{\mu\nu}^t = \eta_{\mu\nu} - u_\mu u_\nu - \frac{\widetilde{P}_\mu \widetilde{P}_\nu}{\widetilde{P}^2},$$

$$\mathcal{M}_{\mu\nu} = -\frac{1}{\sqrt{-2\widetilde{P}^2}} (\bar{u}_\mu P_\nu + \bar{u}_\nu P_\mu), \quad \mathcal{L}_{\mu\nu} = \frac{P_\mu P_\nu}{P^2},$$

with  $\widetilde{P}_\mu = P_\mu - \omega u_\mu$  and  $\bar{u}^\mu = u^\mu - (\omega/P^2)P^\mu$ . Both  $\mathcal{P}_l^{\mu\nu}$  and  $\mathcal{P}_t^{\mu\nu}$  are transverse with respect to  $P^\mu$ , while  $\mathcal{M}^{\mu\nu}$  satisfies a weaker condition  $P_\mu \mathcal{M}^{\mu\nu} P_\nu = 0$ . The scalar functions in (1) are extracted as

$$\pi_l = \mathcal{P}_l^{\mu\nu} \pi_{\mu\nu}, \quad \pi_t = \frac{1}{2} \mathcal{P}_t^{\mu\nu} \pi_{\mu\nu},$$

$$\pi_m = -\mathcal{M}^{\mu\nu} \pi_{\mu\nu}, \quad \tilde{\pi} = \mathcal{L}^{\mu\nu} \pi_{\mu\nu}.$$

Here  $\pi_m$  and  $\tilde{\pi}$  measure the deviation from transversality. In high temperature perturbative QCD (pQCD), the transversality holds only in the temporal axial gauge and Feynman gauge [8]. In general the violation of transversality is however sub-leading in temperature in pQCD and one usually neglects  $\pi_m$  and  $\tilde{\pi}$ .

Now from (1), the most general form of the gluon propagator  $\mathcal{D}_{\mu\nu} = \mathcal{D}_{0,\mu\nu} (1 + \pi_{\mu\nu} \mathcal{D}_{0,\mu\nu})^{-1}$  follows as

$$\mathcal{D}_{\mu\nu} = -\frac{\mathcal{P}_{\mu\nu}^t}{P^2 - \pi_t} - 2 \left[ 2 (P^2 - \pi_l) (\xi^{-1} P^2 - \tilde{\pi}) + \pi_m^2 \right]^{-1}$$

$$\times \left[ (\xi^{-1} P^2 - \tilde{\pi}) \mathcal{P}_{\mu\nu}^l + \pi_m \mathcal{M}_{\mu\nu} + (P^2 - \pi_l) \mathcal{L}_{\mu\nu} \right]. \quad (4)$$

The Slavnov-Taylor identity (STI) in covariant gauge,  $P^\mu \mathcal{D}_{\mu\nu} P^\nu = P^\mu \mathcal{D}_{0,\mu\nu} P^\nu - \xi$ , leaves three independent components in (4). The most general form of the non-perturbative gluon propagator is

$$D_{\mu\nu}^{ab,np}(P) = \mathcal{D}_{\mu\nu}^{ab,\text{exact}}(P) - \mathcal{D}_{\mu\nu}^{ab,\text{pert}}(P)$$

$$= P_{\mu\nu}^l D_l + P_{\mu\nu}^t D_t + \mathcal{M}_{\mu\nu} D_m, \quad (5)$$

in an obvious notation.

The chromoelectric and chromomagnetic condensates are given by the second moment of the nonperturbative gluon propagator [17]

$$\langle \mathcal{E}^2 \rangle_T = -TF_A \int \frac{d^3 k}{(2\pi)^3} D_l(0, k) k^2, \quad (6a)$$

$$\langle \mathcal{B}^2 \rangle_T = 2TF_A \int \frac{d^3 k}{(2\pi)^3} D_t(0, k) k^2, \quad (6b)$$

where  $F_A = N_c^2 - 1$  and  $N_c$  is the number of color. Note that the frequency sum is only restricted to the lowest Matsubara mode ( $k_0 = 0$ ) in the spirit of the plane wave method [17]. This is equivalent to restricting oneself to the most dominant infrared singular sector. Also the

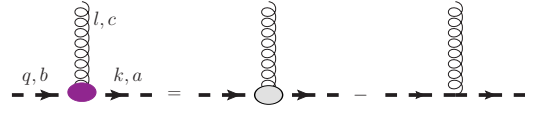


FIG. 1. (color online) The nonperturbative ghost-gluon vertex

lowest Matsubara mode excludes the explicit appearance of  $D_m$  in the gluonic condensates. Similarly, the ghost condensate is given as

$$\langle \bar{\eta} \square \eta \rangle_T = TF_A \int \frac{d^3 k}{(2\pi)^3} G(0, k) k^2, \quad (7)$$

where  $G$  is the nonperturbative ghost propagator.

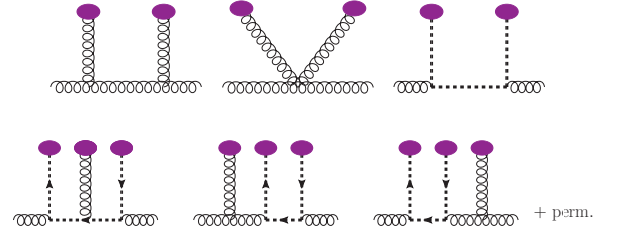


FIG. 2. (color online) Gluon self-energy with gluon condensate (1<sup>st</sup> and 2<sup>nd</sup> diagrams), ghost condensate (3<sup>rd</sup> diagram) and ghost-gluon mixed condensates (4<sup>th</sup>, 5<sup>th</sup>, 6<sup>th</sup> plus their permutation).

Let us note some of the important points considered in our calculations:

1. We work here to the effective order of  $\alpha_s$  in the sense that terms which are higher order in the coupling are related to terms of the order of  $\alpha_s$  through the equations of motion.
2. The gluon condensates are composite operators which do not correspond to any conserved currents and thus, are not renormalizable. Nonetheless, one can extract finite non-renormalizable contributions by combining with other composite operators. The point is that under renormalization the gluonic operators acquire admixtures of certain other operators, e.g., ghost. Therefore, we include the ghost-antighost condensate and two loop contributions involving nonperturbative ghost-gluon vertex as shown in Fig. 1. This fixes uniquely the coefficients of dimension four gluonic condensates [23] in the gluon self-energy as shown in Fig. 2. The moment of the nonperturbative ghost-gluon vertex in Fig. 1 can be obtained as,

$$iT^2 \int \frac{d^3 k}{(2\pi)^3} \frac{d^3 l}{(2\pi)^3} g k_i \Gamma_j^{abc}(k, l) = \frac{f^{abc} \delta_{ij}}{3N_c F_A}$$

$$\times \langle g f^{lmn} \partial_\lambda \bar{\eta}^l A^{\lambda, m} \eta^n \rangle_T. \quad (8)$$

3. At zero temperature, the gluon propagator that includes all possible condensates up to mass dimension four in Fig. 2 satisfies the STI where the ghost and mixed ghost-gluon condensates cancel the longitudinal terms generated by the gluon condensate [15]. However, extending it to finite temperature we find that the STI is not obeyed [24]. This is not unexpected as it is also found in perturbative [8, 21] as well as in nonperturbative calculations [20].

4. We note that the magnetic screening mass depends on the gauge fixing parameter  $\xi$ . On the lattice one can measure various quantities by fixing the gauge. In the same spirit, we also intend, based on OPE with input from LQCD, to estimate screening masses for various gauge choices and compare with those lattice results.

Now, the in-medium propagating modes can be written from (4) as

$$\omega_l(p) \rightarrow P^2 - \pi_l + \frac{\pi_m^2}{2(\xi^{-1}P^2 - \tilde{\pi})} = 0, \quad (9a)$$

$$\omega_t(p) \rightarrow P^2 - \pi_t = 0. \quad (9b)$$

In OPE, the nonperturbative corrections to the polarization tensor are calculated by writing down the full Feynman diagrams and subtracting the equivalent perturbative ones. The soft loop momenta are expanded in powers of external momenta and moments are identified with condensates as described above [25]. This is quite different from the Hard Thermal Loop approximation of pQCD where the polarization operator is saturated by the hard loop momenta ( $\sim T$ ). At finite temperature, the general expressions for scalar functions in the nonperturbative gluon self-energy are obtained by summing all the diagrams in Fig. 2. These functions are quite involved [24] and reveal a rich structure of thermal QCD in a nontrivial background. The screening masses<sup>1</sup> can be extracted from the pole position of the propagator in the spacelike region  $p_0 = 0, p^2 = -M^2$ , where  $M$  is the relevant mass scale. The nonperturbative contribution to various scalar components of the gluon-self energy in the static limit  $p_0 \rightarrow 0$  can be obtained as

$$\pi_l^{np}(0, p) = -\frac{a}{p^2},$$

$$\pi_t^{np}(0, p) = -\frac{b}{p^2} - \frac{R\xi}{p^2} [\langle \bar{\eta}^a \square \eta^a \rangle_T - \langle g f^{abc} \partial_\mu \bar{\eta}^a A^{\mu, b} \eta^c \rangle_T]$$

<sup>1</sup> In Refs. [9, 10] a different prescription was used to extract screening masses from gauge invariant correlators within a dimensionally reduced QCD. This dimensional reduction works at a rather high temperature. A direct comparison of our non-perturbative prescription with those in Refs. [9, 10] may not be justified.

$$+ \frac{R}{p^2} W_M [\langle \bar{\eta}^a \square \eta^a \rangle_T - \langle \mathcal{B}^2 \rangle_T + \langle \mathcal{E}^2 \rangle_T + \dots],$$

$$\pi_m^{np}(0, p) = 0, \quad \tilde{\pi}^{np}(0, p) \neq 0, \quad (10)$$

where,

$$a = \frac{4\pi^2 N_c}{F_A} \left[ \frac{8}{3} \frac{\alpha_s}{\pi} \langle \mathcal{E}^2 \rangle_T + \frac{8}{30} \frac{\alpha_s}{\pi} \langle \mathcal{B}^2 \rangle_T \right], \quad (11a)$$

$$b = \frac{4\pi^2 N_c}{F_A} \left[ W_E \frac{\alpha_s}{\pi} \langle \mathcal{E}^2 \rangle_T - W_B \frac{\alpha_s}{\pi} \langle \mathcal{B}^2 \rangle_T \right]. \quad (11b)$$

Here  $R = \frac{4\pi\alpha_s N_c}{3F_A}$ ,  $W_E = \left(2 + \frac{\xi}{3}\right)$ ,  $W_B = \frac{1}{15}(38 + 9\xi)$  and  $W_M = (2 + \xi)$ . The value of  $a$  obtained here is the same as in Ref. [20]. Condensates appearing in the last two terms in  $\pi_t$  are classical equations of motion for ghost and gluon fields so they vanish. The electric and magnetic screening masses are obtained from

$$m_D^2 = \pi_l^{pert}(0, -m_D^2) + \pi_l^{np}(0, -m_D^2), \quad (12)$$

$$m_m^2 = \pi_t^{pert}(0, -m_m^2) + \pi_t^{np}(0, -m_m^2). \quad (13)$$

To the perturbative order  $\alpha_s$ ,  $\pi_l^{pert}(0, p) = (m_D^{pert})^2 = 4\pi\alpha_s T^2$ , whereas  $\pi_t^{pert}(0, p) = 0$ . Solving (12) and (13), we obtain the values of the screening masses as

$$m_m = b^{\frac{1}{4}}, \quad m_D = \left[ \frac{1}{2} \left\{ (m_D^{pert})^2 + \sqrt{(m_D^{pert})^4 + 4a} \right\} \right]^{\frac{1}{2}}. \quad (14)$$

For numerical evaluations of nonperturbative part in screening masses we use electric and magnetic condensates related to space ( $\Delta_\sigma$ ) and timelike ( $\Delta_\tau$ ) plaquettes measured on lattice for pure  $SU(3)$  gauge theory [13] as

$$\frac{\alpha_s}{\pi} \langle \mathcal{E}^2 \rangle_T = \frac{4}{11} T^4 \Delta_\tau - \frac{2}{11} \langle \mathcal{G}^2 \rangle_0,$$

$$\frac{\alpha_s}{\pi} \langle \mathcal{B}^2 \rangle_T = -\frac{4}{11} T^4 \Delta_\sigma + \frac{2}{11} \langle \mathcal{G}^2 \rangle_0, \quad (15)$$

where  $\langle \mathcal{G}^2 \rangle_0$  is gluon condensate at  $T = 0$  and we take  $\langle \mathcal{G}^2 \rangle_0 / T_c^4 = 2.5$  and the critical temperature,  $T_c = 260$  MeV, for pure  $SU(3)$  gauge theory. The perturbative piece is evaluated using the two loop running coupling constant,

$$\alpha_s(\bar{\mu}) = \frac{4\pi}{\beta_0 \bar{L}} \left[ 1 - \frac{2\beta_1 \ln \bar{L}}{\beta_0^2 \bar{L}} \right], \quad (16)$$

with  $\beta_0 = 11$ ,  $\beta_1 = 51$ ,  $\bar{L} = \ln(\bar{\mu}^2/\Lambda^2)$ . We take  $\bar{\mu} = 2\pi T$  and  $\Lambda = 1.03 T_c$ .

We further note that there is no  $\alpha_s$  dependence of the condensates in (15), i.e., they cannot be expanded as a power series with respect to  $\alpha_s$ , since they are based on non-perturbative LQCD results [17]. Due to the same reason, the parametric power counting hierarchy of scales  $m_D > m_m \sim g^2 T$  is obscure at low temperature where the condensates might provide plausible explanation for the magnetic mass.

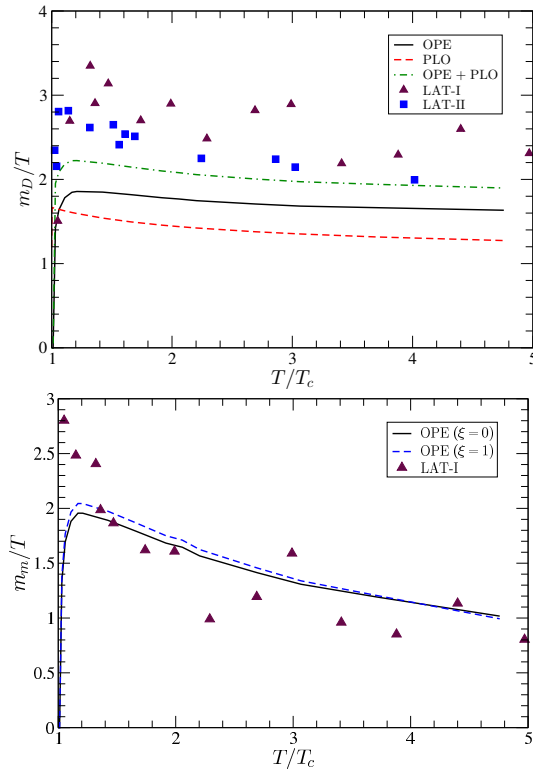


FIG. 3. (color online) Temperature variation of electric (upper panel) and magnetic (lower panel) screening masses. The present investigation is represented by OPE with two different gauge fixing parameter  $\xi$  whereas LQCD data are represented by LAT-I [3] and LAT-II [4]. PLO is perturbative leading order.

The electric and magnetic screening masses so obtained are delineated in Fig. 3. We find that the nonperturbative contribution in  $m_D$  dominates over the perturbative leading order (PLO) contribution for the temperature range we considered. The complete electric screening mass, including perturbative and nonperturbative contributions, falls short of lattice data but is still rather close to it. On the other hand, the magnetic screening is purely nonperturbative in nature and agrees relatively well with lattice data. As seen the magnetic mass is dependent upon the gauge fixing parameter and we have chosen Landau ( $\xi = 0$ ) and Feynman ( $\xi = 1$ ) gauge. The weak gauge dependence found here is in agreement with that of Ref. [3] for a similar choice of gauge fixing.

In summary, we have for the first time computed the nonperturbative contribution to both chromoelectric and chromomagnetic screening masses using OPE in a gluonic plasma. In particular, the magnetic screening mass is in relatively good agreement with the LQCD data whereas that of electric screening exhibits some discrepancy. The OPE electric screening mass is about 20% below the LQCD data points, which, however, show a rather large spread. The knowledge of these quantities sets the dynamical length scale and provides us with the active de-

grees of freedom in a hot QCD plasma. These results may be useful input to calculate various thermodynamic quantities, spectral properties and for the phenomenology of jet quenching [26], quarkonium suppression [27] in hot QCD matter produced in relativistic heavy-ion collision experiments.

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